

## The Concept of Complex Numbers, Chained Fractions

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### Article Information

**Received:** December 25, 2022

**Accepted:** January 26, 2023

**Published:** February 28, 2023

**Keywords:** arithmetic operations, complex numbers, chain fractions, natural numbers, mathematical properties, etc, teacher.

### ABSTRACT

*In this article, the concept of complex numbers calculated from mathematical properties and the concepts of chain fractions are explained and detailed examples are presented.*

### Introduction

When working with real numbers, it was said that the square of any real number different from zero is positive. But it is necessary to deal with numbers whose square is negative. Naturally, such numbers are not real numbers. Using the geometric form of a complex number and the above definitions, we can make the following points. Pure abstract numbers are defined on the abstract axis  $z=0+ib$ , real numbers  $z=a$  on the real axis. The complex numbers  $z=a+ib$  and  $z=a-ib$  are symmetrical about the axis of real numbers. Opposite complex numbers are located symmetrically with respect to the origin of the coordinate. In the complex plane, the position of the point  $z$  is determined not only in the Cartesian coordinate system  $x, y$ , but also in the polar coordinates  $r, \varphi$ , where  $r$  is the distance from point 0 to point  $z$  and  $\varphi$  is the positive direction of the real axis the angle between the angle and the  $z$  vector. If the direction is taken counterclockwise, the value of the angle is positive, if it is taken clockwise, it is negative. This angle  $\varphi$  is called the argument of the complex number and is denoted by  $\text{Arg}z$ . For the number  $z$ , the argument is not defined, therefore,  $z$  is considered in the arguments related to the following arguments. Write the following set of points of the complex plane using the inequality:

1. The half-plane to the right of the abstract axis.
2. The first quarter.
3. A half-plane consisting of points above the real axis and at a distance of not less than 2 from the real axis.
4. A corridor consisting of points at a distance smaller than one from an abstract axis.
5. A semicircle with a radius of 1 centered at the left of the abstract axis.

6. A half-plane consisting of points located below the real axis and at a distance greater than three units from the real axis;
7. A corridor consisting of points whose distance to the real axis is less than one;
8. A semicircle with a center point, a radius equal to 1 below the true axis (does not rotate);
9. To the general center; circle between circles with radii equal to 1 and 2 (without circles).

$$\lim_{n \rightarrow \infty} z_n = \lim_{n \rightarrow \infty} \left( \frac{2n+5}{3n+4} + i \frac{4n-7}{5n-2} \right) = a = \frac{2}{3} + i \frac{4}{5}.$$

The value of the limit takes part in the definition of the sequence limit. But it is not always easy to find the value of the limit for any (even approximating) sequence. Therefore, in practice, it is necessary to know the sign in which the value of the limit does not participate in the convergence of the sequence. This symptom consists of the Cauchy criterion. It is necessary to prove the existence of a limit of the sequence of modules and arguments (in precision up to integer multiples) and calculate these limits. For the derivatives of addition, subtraction, multiplication and division, the formulas in real analysis are appropriate. However, not all the properties of the differentiable real function are appropriate for the complex-valued function. In particular, Roll's and Lagrange's theorems are not always appropriate for complex-valued functions.

$$\frac{9}{7} = 1 + \frac{2}{7} = 1 + \frac{1}{\frac{7}{2}} = 1 + \frac{1}{3 + \frac{1}{2}} = 1 + \frac{1}{3 + \frac{1}{1 + \frac{1}{1}}}.$$

It turns out, however, that fractions of this form, called "continued fractions", provide much insight into many mathematical problems, particularly into the nature of numbers. Continued fractions were studied by the great mathematicians of the seventeenth and eighteenth centuries and are a subject of active investigation today. Nearly all books on the theory of numbers include a chapter on continued fractions, but these accounts are condensed and rather difficult for the beginner. The plan in this book is to present an easygoing discussion of simple continued fractions that can be understood by anyone who has a minimum of mathematical training. It goes without saying that one should not "read" a mathematics book. It is better to get out pencil and paper and rewrite the book. A student of mathematics should wrestle with every step of a proof; if he does not understand it in the first round, he should plan to return to it later and tackle it once again until it is mastered. In addition he should test his grasp of the subject by working the problems at the end of the sections. These are mostly of an elementary nature, closely related to the text, and should not present any difficulties.

We should notice that in equation (1.10) the number 2.29 is the largest multiple of 29 that is less than 67, and consequently the remainder (in this case the number 9) is necessarily a number 20 but definitely  $< 29$ . Next consider equation (1.11). Here 3.9 is the largest multiple of 9 that is less than 29. The remainder, 2, is necessarily a number 20 but  $< 2$ . Finally, we cannot go beyond equation (1.12), for if we write

$$\frac{9}{2} = 4 + \frac{1}{2} = 4 + \frac{1}{1},$$

We observe, in this example that in the successive divisions the remainders 9, 2, 1 are exactly determined non-negative numbers each smaller than the corresponding divisor. Thus the remainder 9 is less than the divisor 29, the remainder 2 is less than the divisor 9, and so on. The

remainder in each division becomes the divisor in the next division, so that the successive remainders become smaller and smaller non-negative integers. Thus the remainder zero must be reached eventually, and the process must end. Each remainder obtained in this process is a unique non-negative number. For example, can you divide 67 by 29, obtain the largest quotient 2, and end up with a remainder other than 9? This means that, for the given fraction  $\frac{67}{29}$ , our process yields exactly one sequence of remainders. Clearly the expansion  $[2, 3, 4, 1, 11]$  can be changed back to its original form  $[2, 3, 4, 21]$ . We shall see in the more general discussion which follows that this is the only way we can get a "different" expansion.

So far we have introduced the terminology peculiar to the study I. of continued fractions and have worked with particular examples. But to make real progress in our study we must discuss more general results. Working with symbols instead of with actual numbers frees the mind and allows us to think abstractly. Thus, while our first theorem merely expresses in general terms what we did in the worked examples, once this has been accomplished a host of other ideas quickly follows. **THEOREM 1.1.** Any finite simple continued fraction represents a rational number. Conversely, any rational number  $p/q$  can be represented as a finite simple continued fraction; with the exceptions to be noted below, the representation, or expansion, is unique. **PROOF.** The first sentence in this theorem is quite clear from what we have explained in our worked examples, for if any expansion terminates we can always "back track" and build the expansion into a rational fraction.

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