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On Zero Divided by Zero

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ABSTRACT

In this study, we define the concept of inverse two operations and show that, i) Let $(G,*)$ *be a group, for all* $a,b \in G$, *then* $a *^{-1} b = a * b^{-1}$
ii) Let $(G,*)$ *be a group,* $e \in G$ *such that* $a * e = e * a = a$ *for all* $a \in G$, *then* $a *^{-1} a = e$ *for all* $a \in G$.

iii) Let $(G,*,\#)$ *be a ring,* $e \in G$ *such that* $a * e = e * a = a$ *for all* $a \in G$ *then* $e \#^{-1} e = x$ *for all* $x \in G$.

iv) Let $\ddot{O} = \{a : a \times 0 = 0\}$, $+$ *be an ordinary addition operation,* $\times$ *be an ordinary multiplication, then* $(\ddot{O}, +, \times)$ *be a ring,* $0 \in \ddot{O}$ *such that* $a + 0 = 0 + a = a$, $\div$ *be an inverse operation for* $\times$, *and* $0 \div 0 \equiv \ddot{O}^{or}$, *i.e* $0 \div 0 = a$, *for all* $a \in \ddot{O}$.

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1. Introduction

Let G be a non empty set , then $(G,*)$ is called a mathematical system if $*$ is binary operation defined on G, let $(G,*)$ be a mathematical system, then $(G,*)$ is called a semi group if $*$ is associative i.e $(a * b) * c = a * (b * c)$ for all $a,b,c \in G$, let $(G,*)$ be a semi group, then $(G,*)$ is called a group if there exist $e \in G$ such that $a * e = e * a = a$ for all $a \in G$ and for all $a \in G$, there exist $a^{-1} \in G$ such that $a * a^{-1} = a^{-1} * a = e$, let $(G,*)$ be a group, then $(G,*)$ is called a commutative group if $a * b = b * a$ for all $a,b \in G$ [1], [3]. Let $(G,*)$ be a commutative group and $(G,\#)$ be a semi group, then $(G,*,\#)$ is called a ring if $a\#(b * c) = (a\#b) * (a\#c)$ and $(a * b)\#c = (a\#c) * (b\#c)$, for all $a,b,c \in G$ [2].

2. Inverse two operations:

In this section we present the concept of inverse two operations and we study the properties of them.

We begin with the following definition:

Definition 2.1:

Let $(G,*)$ is mathematical system, then $*^{-1}$ is called inverse operation for $*$, also $*$ is called inverse operation for $*^{-1}$, i.e $*$ and $*^{-1}$ are inverse two operations, if $a * b = c$ then $a = c *^{-1} b$ and if $a = c *^{-1} b$ then $a * b = c$ (i.e $a * b = c$ iff $a = c *^{-1} b$) for all $a,b,c \in G$.

Example 2.2:

$(R, +)$is mathematical system such that R is the set of real nambers and $+$ is ordinary addition operation, then ordinary subtraction operation $(-)$ is inverse operation for $+$, since $a + b = c$ iff $a = c - b$ for all $a, b, c \in R$.

Example 2.3:

$(R, \times)$is mathematical system such that R is the set of real nambers and $\times$ is ordinary multiplication operation, then ordinary division operation $(\div)$ is inverse operation for $\times$, since $a \times b = c$ iff $a = c \div b$ for all $a, b, c \in R$.

Remark 2.4:

Let $(G, *)$ is mathematical system, $*^{-1}$ be an inverse operation for $*$, then for all $a, b, c \in G$

i) $a * b * c = (a * b) * c$
ii) $a *^{-1} b *^{-1} c = (a *^{-1} b) *^{-1} c$
iii) $a * b *^{-1} c = (a * b) *^{-1} c$
iv) $a *^{-1} b * c = (a *^{-1} b) * c$

Lemma 2.5:

Let $(G, *)$ is mathematical system, $*^{-1}$ be an inverse operation for $*$, then for all $a, b \in G$
i) $a * b *^{-1} b = a$
ii) $a *^{-1} b * b = a$

Proof.

i) Let $a * b = c$ ... (1) by Definition 2.1, we get $a = c *^{-1} b \ldots (2)$

From (1) and (2), we get $a = (a * b) *^{-1} b$

By Remark 2.4 (iii), we get $a * b *^{-1} b = a$

ii) Let $c * b = a \ldots (1)$ such that $c \in G$ by Definition 2.1, we get

$c = a *^{-1} b \ldots (2)$ from (1) and (2), we get $(a *^{-1} b) * b = a$,

by Remark 2.4 (iv), we get $a *^{-1} b * b = a$.

Lemma 2.6:

Let $(G, *)$ is mathematical system, $*^{-1}$ be an inverse operation for $*$, then for all $a, b, c \in G$

i) if $a * c = b * c$ then $a = b$

ii) if $a *^{-1} c = b *^{-1} c$ then $a = b$

Proof.

i) $a * c = b * c$

by Definition 2.1, we get $a = b * c *^{-1} c$, by Lemma 2.5 (i), we get $a = b$

ii) $a *^{-1} c = b *^{-1} c$

by Definition 2.1, we get $a = b *^{-1} c * c$, by Lemma 2.5 (ii), we get $a = b$

Lemma 2.7:

Let $(G, *)$ be a semi group, $*^{-1}$ be an inverse operation for $*$, then

$(a * b) *^{-1} c = a * (b *^{-1} c)$

Proof.

Let $(a * b) *^{-1} c = x$ by D efinition 2.1,

we get $a * b = x * c \dots (1)$

Let $b *^{-1} c = y$ by Definition 2.1, we get $b = y * c \dots (2)$

From (1) and (2), we get $a * (y * c) = x * c$, $(G,*)$ be a semi group i.e $*$ is associative, weget $(a * y) * c = x * c$

By Lemma 2.6 (i), we get $a * y = x$

i.e $a * (b *^{-1} c) = (a * b) *^{-1} c$

Lemma 2.8:

Let $(G,*)$ be a semi group, $*^{-1}$ be an inverse operation for $*$, then

$a *^{-1} (b * c) = a *^{-1} c *^{-1} b$.

Proof.

Let $a *^{-1} (b * c) = x$ by Definition 2.1, we get $a = x * (b * c)$, $(G,*)$ be a semi group i.e $*$ is associative, we get $a = (x * b) * c$ by Definition 2.1, we get $a *^{-1} c = x * b$, also $a *^{-1} c *^{-1} b = x$

i.e $a *^{-1} (b * c) = a *^{-1} c *^{-1} b$

Theorem 2.9:

Let $(G,*)$ be a group, $*^{-1}$ be an inverse operation for $*$, then $a *^{-1} b = a * b^{-1}$, such that $b * b^{-1} = b^{-1} * b = e$ and $a * e = e * a = a$, for all $a \in G$.

Proof.

Let $a *^{-1} b = x$ by Definition 2.1, we get $a = x * b$ ... (1)

Let $a * b^{-1} = y$, $(a * b^{-1}) * b = y * b$, $(G,*)$ be a semi group i.e $*$ is associative, we get $a * (b^{-1} * b) = y * b$, $a * e = y * b$, $a = y * b$ ... (2)

From (1) and (2), we get $x * b = y * b$ by Lemma 2.6 (i), we get $x = y$ i.e $a *^{-1} b = a * b^{-1}$

Theorem 2.10:

Let $(G,*)$ be a group, $*^{-1}$ be an inverse operation for $*$, then $a *^{-1} a = e$ such that $a * e = e * a = a$ for all $a \in G$.

Proof.

Let $a *^{-1} a = x$ by Definition 2.1, we obtain $a = x * a$, but $a = e * a$, we get $x * a = e * a$ by Lemma 2.6 (i), we obtain $x = e$ i.e $a *^{-1} a = e$.

(Or by Theorem 2.9 $a *^{-1} a = a * a^{-1} = e$)

Lemma 2.11:

Let $(G,*)$ be a group, $e \in G$ such that $a * e = e * a = a$ for all $a \in G$, if $b * b = b$, $b \in G$, then $b = e$.

Proof.

$b * b = b$, $b * b = e * b$ by Lemma 2.6 (i), we get $b = e$.

Lemma 2.12:

Let $(G,*)$ be a group, $e \in G$ such that $a * e = e * a = a$ for all $a \in G$, then $e * e = e$

Proof.

Let $x = e * e$, $x * e = e * e$ by Lemma 2.6 (i), we get $x = e$, i.e $e * e = e$.

Lemma 2.13:

Let $(G,*,\#)$ be a ring, $e \in G$ such that $a * e = e * a = a$ for all $a \in G$ then $x \# e = e$ for all $x \in G$.

Proof.

Let $x \in G$, by Lemma 2.12 $x\#e = x\#(e * e)$, $(G,*,\#)$ be a ring, i.e $x\#(e * e) = (x\#e) * (x\#e)$, we get $x\#e = (x\#e) * (x\#e)$, by Lemma 2.11 $x\#e = e$.

Theorem 2.14:

Let $(G,*,\#)$ be a ring, $e \in G$ such that $a * e = e * a = a$ for all $a \in G$, then $e \#^{-1} e = x$ for all $x \in G$.

Proof.

Let $x \in G$, by Lemma 2.13 $x\#e = e$, $(G,*,\#)$ be a ring, i.e $(G,\#)$ is mathematical system, by Definition 2.1, we get $x = e \#^{-1} e$.

Example 2.15:

Let $\mathbb{C}$ be a complex numbers set, $+$ be an ordinary addition operation, $\times$ be an ordinary multiplication operation, then $(\mathbb{C}, +, \times)$ be a ring , $0 \in \mathbb{C}$ such that $a + 0 = 0 + a = a$, $\div$ be an inverse operation for $\times$ (Example 2.3), by Theorem 2.14, we get $0 \div 0 = x$ for all $x \in \mathbb{C}$.

Definition 2.16:

Let A be a non empty set, then $A^{or} = a$, for all $a \in A$, i.e if $A = \{a, b, c, d, \dots \dots\}$, then $A^{or} \equiv$ the concept $(a \ or \ b \ or \ c \ or \ d \ or \dots)$ Note the symbol $(\equiv)$ means equivalent i.e A^{or} is the same concept $(a \ or \ b \ or \ c \ or \ d \ or \dots)$

Example 2.17:

Let $A = \{1,2\}$, then $A^{or} = 1$ and $A^{or} = 2$, since $\{1,2\}^{or} \equiv (1 \ or \ 2)$ Note $(1 \ or \ 2) = 1$ correct phrase, $(1 \ or \ 2) = 2$ correct phrase.

Remark 2.18:

$\{a\}^{or} = a \quad \forall a \in \mathbb{C}$.

Remark 2.19:

Let $n \in Z^{+}$, then $(x)^{\frac{1}{n}} = y$ iff $x = y^{n}$ i.e $(x)^{\frac{1}{n}} = y$ for all $y \in \{z : z^{n} = x\}$, $(x)^{\frac{1}{n}} \equiv \{z : z^{n} = x\}^{or}$

Example 2.20:

$(25)^{\frac{1}{2}} \equiv \{5, -5\}^{or}$ but $\sqrt{25} = 5$ or $\sqrt{25} \equiv \{5\}^{or}$

i.e $(25)^{\frac{1}{2}} = \sqrt{25}$ and $(25)^{\frac{1}{2}} \neq \sqrt{25}$ Since $5 \ or \ -5 \neq \sqrt{25}$ correct phrase

i.e the concept $(25)^{\frac{1}{2}}$ is not equivalent to the concept $\sqrt{25}$.

Example 2.21:

$\int dx = x + c$ such that c is constant, then $\int dx \equiv x + R^{or} \equiv (x+R)^{or}$ Such that R is real numbers, $x + R = \{ y : y = x + r \,,\; r \in R \}$

Remark 2.22:

Let $\ddot{O} = \{ a : a \times 0 = 0 \}$, $+$ be an ordinary addition operation $\times$ be an ordinary multiplication, then $(\ddot{O} , + , \times)$ be a ring, $0 \in \ddot{O}$ such that $a + 0 = 0 + a = a$, $\div$ be an inverse operation for $\times$, by Theorem 2.14 and Definition 2.16, we get $0 \div 0 \equiv \ddot{O}^{or}$, i.e $0 \div 0 = a$, for all $a \in \ddot{O}$. Note $\mathbb{C}$ subset of $\ddot{O}$.

Remark 2.23:

Let $y = 1$ is equation of $\overleftrightarrow{L}$, then $(r,1) \in \overleftrightarrow{L}$,$\forall\, r \in R$, R is real numbers, i.e Set all Point $\overleftrightarrow{L} = \{(r,1) : r \in R\}$.

Let $y = \frac{x}{x}$ is equation of geometric shape T, if $x \neq 0$, then $y = 1$, if $x = 0$, then $y = R^{or}$, i.e $y = a$ for all $a \in R$, we get $(b,1) \in T$,$\forall\; b \in R \,, b \neq 0$ and $(0,a) \in T$,$\forall\; a \in R$, Set all Point $T = \{(0,a) : a \in R\} \cup \{(b,1) : b \in R \,, b \neq 0\}$, we conclude the geometric shape T is $\overleftrightarrow{L} \cup y - axis$.

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